1 Generationg Pythagrean Tree

The construction of the Pythagorean tree (Figure 1) starts from one square. Construct a right triangle whose hypotenuse is the top edge of the square. Construct two squares along each of the other two sides of this triangle. Repeat this construction recursively on each of the two new squares.
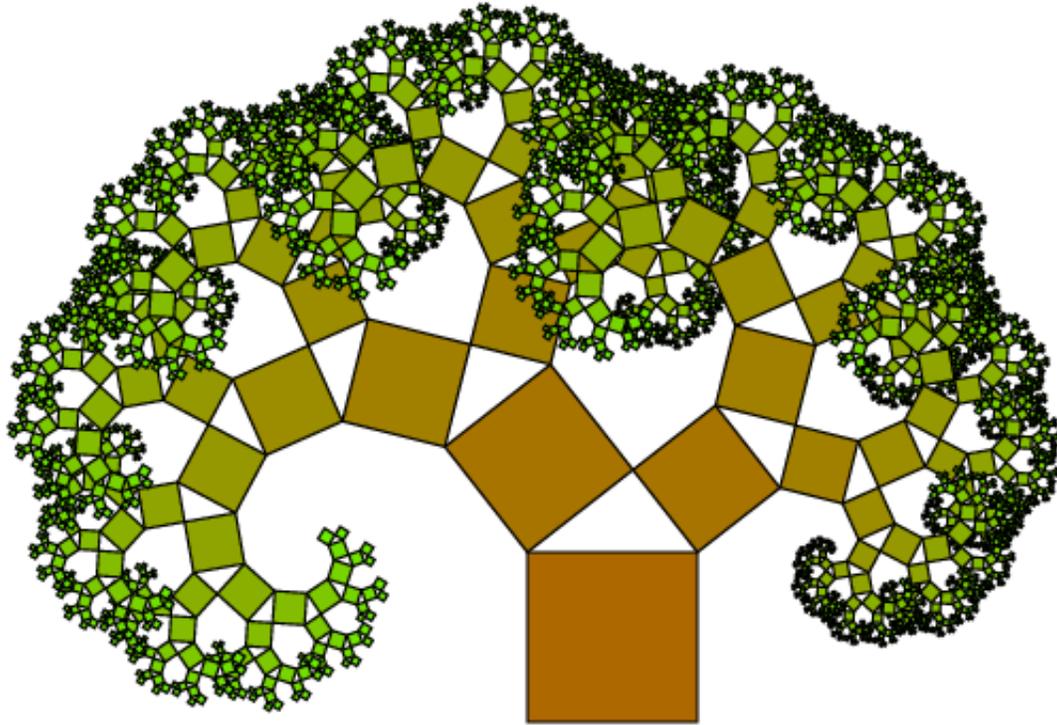

Figure 1: A Pythagorean Tree

1.1 Iterated Function of Pythagorean Tree

Let the initial square be a unit square with the lower left corner at the origin, O. Let α be the angle show in Figure 2. Then on the next iteration there are two squares, one being the on the left with side $\cos \alpha$ and one being on the right with side $\sin \alpha$. The left side square is rotated counter-clockwise by α and translated by $(0, 1)'$ from the original unit square. The right side square is rotated $90° - \alpha$ clockwise and translated by $(\cos^2 \alpha, 1 + \cos \alpha \sin \alpha)'$ from the original unit square. Therefore the two iterated functions corresponding to the left side and right side squares are,

$$f_1(X) = \cos \alpha \left[\begin{array}{cc} \cos \alpha & -\sin \alpha \\ \sin \alpha & \cos \alpha \end{array} \right] X + \left[\begin{array}{c} 0 \\ 1 \end{array} \right]$$

$$f_2(X) = \sin \alpha \left[\begin{array}{cc} \sin \alpha & \cos \alpha \\ -\cos \alpha & \sin \alpha \end{array} \right] X + \left[\begin{array}{c} \cos^2 \alpha \\ 1 + \cos \alpha \sin \alpha \end{array} \right]$$

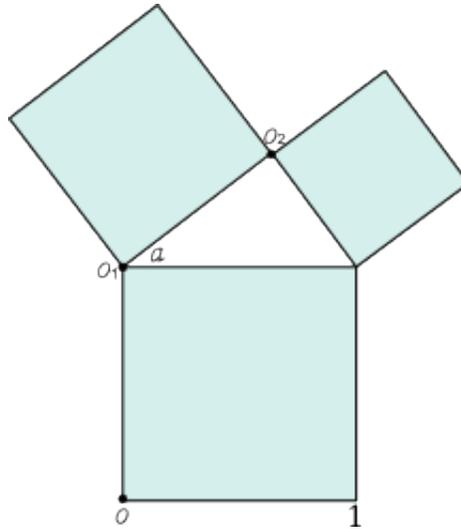α is taken to be $38°$ in the final logo tree.

Figure 2: First Iteration of Pythagorean Tree

2   Generating the Shade

Before we get to the generating of the shade (Figure 3), first let's consider generating triangles inside a square (Figure 4a). The triangles are constructed from the outside inward, first the outside one, then the one next to it, . . ., the last one is the center one. The $x-$coordinates of the two bottom vertices of the triangle move toward the center and eventually meet or almost meet each other; The $y-$coordinates of the two bottom vertices of the triangle move upward with only half the step of their $x-$coordinates and eventually land at $1/4$ of the square from the bottom; The top vertex of the triangle moves downward the same step as the $x-$coordinates of the bottom two vertices and eventually moves to the center of the square.

Now consider filling in each triangle with color. Give a color for the outermost triangle and a color for the innermost triangle, fill in the triangles in between smoothly. Figure 4b shows this color transition from black to white.

To make the color pattern interesting, now consider rotating each triangle around the center of the bounding square while filling in its color. The rotation angle is set to be the same for each triangle going to its succeeding triangle, and when the last triangle is done, the total rotation is $360°$.
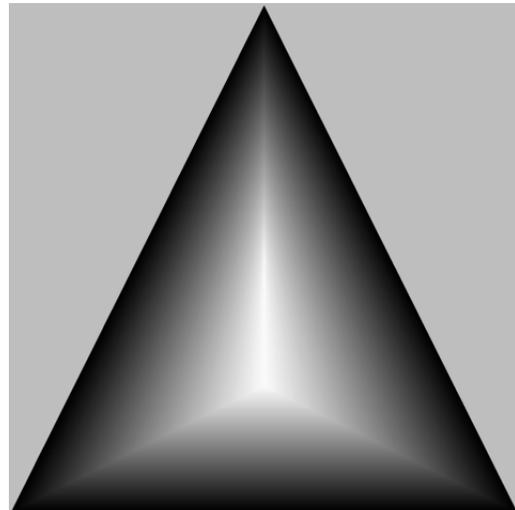
By varying the number of triangles, we can get interesting results. Figure 5 shows some of the results.

Figure 3: Shade Used in Logo
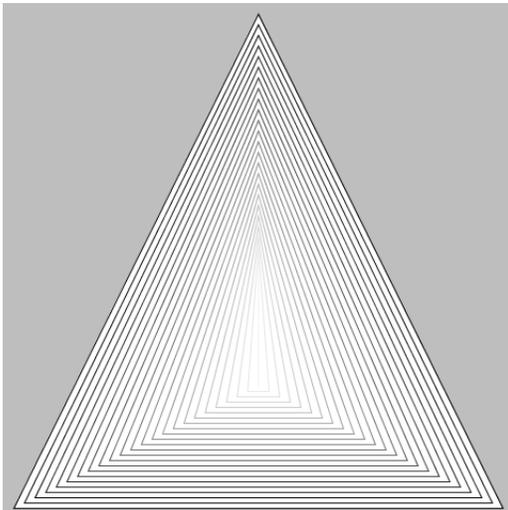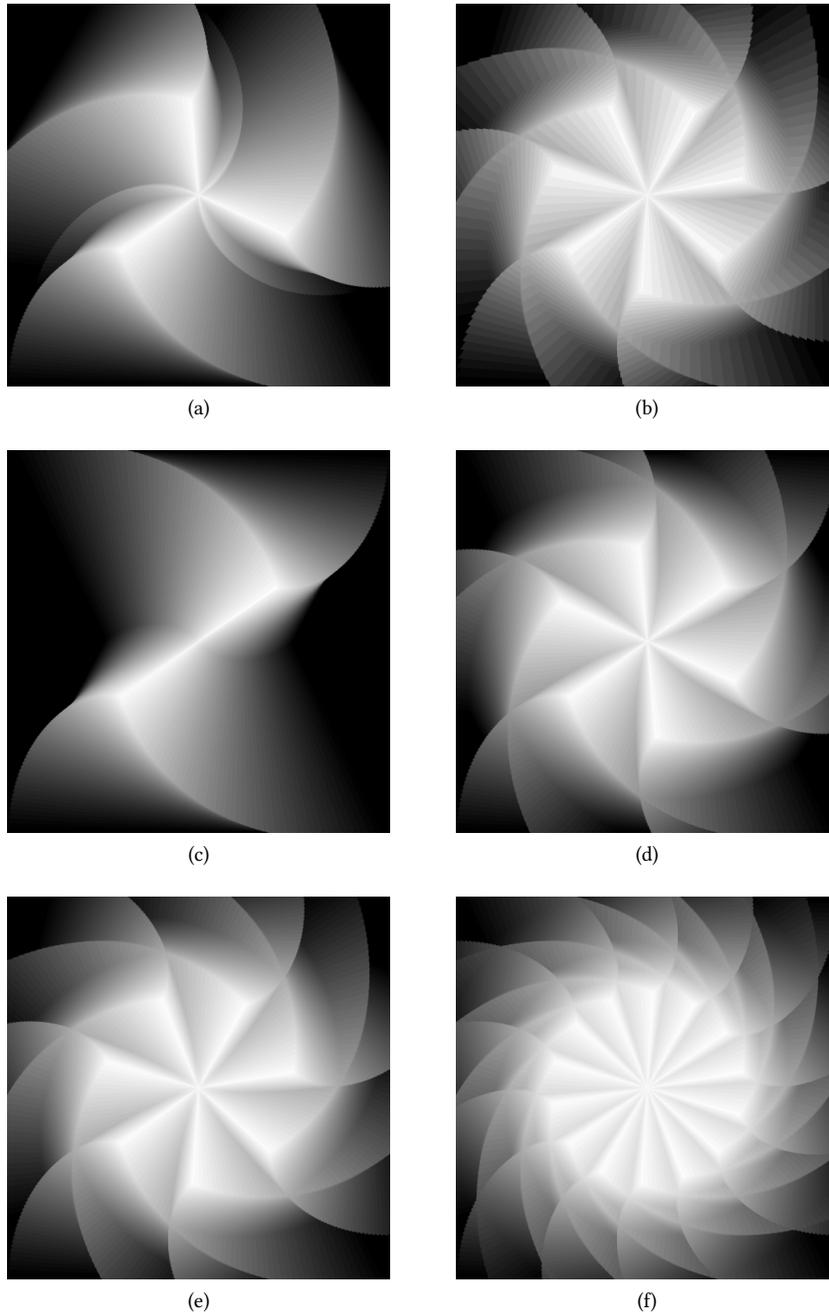

(a) Triangle Zoom

(b) Triangle Blur

Figure 4: Construction of Triangles and Triangle Blur

Figure 5: Some Shading Results